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Decoherence of electron beams by electromagnetic field fluctuations

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Abstract

Electromagnetic field fluctuations are responsible for the destruction of electron coherence (dephasing) in solids and in vacuum electron beam interference. The vacuum fluctuations are modified by conductors and dielectrics, as in the Casimir effect, and hence, bodies in the vicinity of the beams can influence the beam coherence. We calculate the quenching of interference of two beams moving in vacuum parallel to a thick plate with permittivity $\epsilon(\omega) = \epsilon_0 + i4\pi\sigma/\omega$. In the case of an ideal conductor or dielectric ($|\epsilon| = \infty$) the dephasing is suppressed when the beams are close to the surface of the plate, because the random tangential electric field E_t, responsible for dephasing, is zero at the surface. The situation is changed dramatically when ϵ_0 or σ is finite. In this case there exists a layer near the surface, where the fluctuations of E_t are strong due to evanescent near fields. The thickness of this near-field layer is of the order of the wavelength in the dielectric or the skin depth in the conductor, corresponding to a frequency which is the inverse electron time of flight from the emitter to the detector. When the beams are within this layer their dephasing is enhanced and slow enough electrons can be even stronger than far from the surface.

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1. Introduction

Quantum electromagnetic (EM) field fluctuations are well known as being responsible for the Casimir forces, see for example [1]. Less known is the role of these fluctuations in the destruction of electron coherence. In weak localization phenomena in solids EM fluctuations are one of the dephasing mechanisms of conduction electrons [2], see also [3]. The interference of vacuum electron beams, observed experimentally [4, 5], is also quenched by EM fluctuations [6], see also [7]. These two papers consider EM fluctuations in a vacuum or when ideal conductors are present in the vicinity of the beams. The role of dissipation was discussed

in [8]. Based on physical arguments, the decoherence was related to the deceleration of an electron from the beam due to the Ohmic dissipation of the current produced in the metal by the image charge.

The aim of this paper is to extend the calculations of [6, 7] to the case where the beams are close to *dissipative* bodies and to consider in detail the experiment geometry when two interfering beams move in vacuum parallel to a thick infinite plate with permittivity $\epsilon(\omega) = \epsilon_0 + \mathrm{i}4\pi\sigma/\omega$. Calculations of the dephasing factor in this geometry demonstrate the crucial role of dissipation. If the plate is an ideal conductor, $\sigma = \infty$, the fluctuations of the tangential electric field E_t, which are responsible for beam dephasing in this geometry, are suppressed near the plate surface because of the boundary condition $E_t = 0$ at the surface. However, when σ is finite, very strong fluctuations of E_t exist near the plate surface, within a layer of the order of the skin depth. These near-field fluctuations dramatically enhance the beam dephasing. Unexpectedly, a similar effect also exists near a lossless dielectric with high permittivity, $\sigma = 0$, $\epsilon_0 \gg 1$, within a layer of the order of the wavelength in the dielectric.

The paper is organized as follows. In section 2 we present the dephasing factor e^{-K} in terms of the EM field correlator $\mathcal{D}$ in the case of no dissipation, equation (7), and give reasons why the quantum Langevin equation for the EM field has to be used when dissipation is present. In section 3 we derive the Langevin equation and prove that the expression of K in terms of $\mathcal{D}$ is valid in the case of dissipation too. In this section we also present the relation between $\mathcal{D}$ and the EM field retarded Green function $\mathcal{G}$, which is used to calculate $\mathcal{D}$. The above-mentioned special geometry is considered in section 4, where K is given as an integral, equation (32), over wave vectors and frequencies, containing the spectral density of the EM field fluctuations $\langle E_t E_t \rangle_{\mathbf{k}\omega}$, and the spectral density $|(\mathbf{l})_{\mathbf{k}\omega}|^2$ of the EM field radiated by the beam electrons. $\langle E_t E_t \rangle_{\mathbf{k}\omega}$ is calculated in section 5 and section 6, where equations (53) and (54) demonstrate the enhancement of fluctuations due to near fields. In section 7 we present a model for $|(\mathbf{l})_{\mathbf{k}\omega}|^2$ and calculate K explicitly as a function of the distance of the beams from the plate d and the electron velocity v (see equations (61), (62) and (63) and the text which follows). It turns out that the dephasing enhancement due to near fields is appreciable when v and σ are not very large. In section 8 we discuss the relation between beam dephasing and beam EM radiation. The appendix contains some calculation details.

2. Beam dephasing

If one ignores the interaction of the beam electrons with the EM field, the number of electrons measured in the interference experiment is $n = |\psi_1|^2 + |\psi_2|^2 + 2\,\mathrm{Re}(\psi_1\psi_2^*)$, where ψ_1 and ψ_2 are the wavefunctions corresponding to the coherent motion of the electrons in beams 1 and 2, and n is calculated at the detector position. The interaction with the EM field does not affect the squares $|\psi_1|^2$ and $|\psi_2|^2$ (since it does not change the number of electrons in the beams), but the product $\psi_1\psi_2^*$, responsible for the interference pattern, is multiplied by a factor $\mathrm{e}^{\mathrm{i}\phi}\,\mathrm{e}^{-K}$ with real ϕ and positive K. The first factor only shifts the interference pattern in space, while the second one reduces the amplitude of the interference oscillations (compared to the background $|\psi_1|^2 + |\psi_2|^2$) and describes dephasing.

To calculate the strength of dephasing we use the 'trace of the environment' picture [3]. At $t = t_0$, when the electron is emitted from the source, the environment is in state $|t_0\rangle$. While moving, the electron interacts with the environment and perturbs its state. When the electron moving in beam 1 arrives at the detector at time t_1, the environment evolves due to this interaction to state $|t_1\rangle$. In a similar way one defines the state $|t_2\rangle$. According to the 'trace of the environment' picture $\mathrm{e}^{\mathrm{i}\phi}\,\mathrm{e}^{-K} = \langle t_2|t_1\rangle$.

One can present the final states of the environment in terms of evolution operators,

$$|t_1\rangle = U_1|t_0\rangle \qquad U_1 = \mathcal{T}\exp\left[-\frac{\mathrm{i}}{\hbar}\int_{t_0}^{t_1}\mathrm{d}t\; H_1(t)\right] \tag{1}$$

where $\mathcal{T}$ means time ordering and $H_1(t)$ is the interaction of the electron in beam 1 with the environment. $H_1(t)$ is in the interaction representation, i.e. sandwiched with evolution exponents $\exp[-\mathrm{i}\mathcal{H}t/\hbar]$ containing the beam electron Hamiltonian and the environment Hamiltonian. In a similar way one defines U_2 in terms of $H_2(t)$ and finds $\langle t_2|t_1\rangle = \langle t_0|U_2^{-1}U_1|t_0\rangle$. For an EM environment, choosing a gauge with zero scalar potential, we have

$$H_1(t) = -\frac{1}{c}\int \mathrm{d}\mathbf{r}\,\mathbf{j}_1(\mathbf{r},t)\mathbf{A}(\mathbf{r},t) \tag{2}$$

where $\mathbf{j}_1$ is the electron current density operator for the electron in beam 1 (sandwiched with the evolution exponents containing the beam electron Hamiltonian) and $\mathbf{A}$ is the vector potential operator (sandwiched with the evolution exponents containing the EM environment Hamiltonian). $H_2(t)$ is defined similarly with the current $\mathbf{j}_2$.

In this approach one assumes that at the initial moment t_0 the electron source and the environment are uncorrelated. It is also assumed that the renormalization of the bare electron parameters due to the interaction with the EM environment [7] does not influence substantially the dephasing phenomena.

To proceed we assume, following [6], that the current is a classical quantity. When there is no dissipation in the EM environment, its Hamiltonian is simply the EM field Hamiltonian and the EM field can be quantized expanding it in normal modes. It is well known that in this case the commutator $[\mathbf{A}(\mathbf{r},t),\mathbf{A}(\mathbf{r}',t')]$ is an *imaginary c-number*, and due to the classical nature of the currents $\mathbf{j}_1$ and $\mathbf{j}_2$ the commutators of $H_1(t)$ and $H_2(t')$ have the same property. Because of this property the time ordering affects only the phase of the evolution operators [9], and one can obtain

$$U_1 = \mathrm{e}^{\mathrm{i}\phi_1}V_1 \qquad V_1 = \exp\left[-\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{\infty}\mathrm{d}t\; H_1(t)\right] \tag{3}$$

if H_1 is defined to be zero for $t < t_0$ and for $t > t_1$. The phase ϕ_1 contains the commutator $[H_1(t), H_1(t')]$. Defining $H_2(t)$ in a similar way, we have

$$U_2^{-1}U_1 = \mathrm{e}^{\mathrm{i}(\phi_1-\phi_2)}V_2^{-1}V_1 = \mathrm{e}^{\mathrm{i}(\phi_1-\phi_2)}\,\mathrm{e}^{\mathrm{i}\chi}\exp\left[-\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{\infty}\mathrm{d}t\;(H_1(t)-H_2(t))\right] \tag{4}$$

where the additional phase χ contains the commutator $[H_1(t), H_2(t')]$. Averaging this over $|t_0\rangle$ we find

$$\langle t_2|t_1\rangle = \mathrm{e}^{\mathrm{i}\phi}\,\mathrm{e}^{-K} = \mathrm{e}^{\mathrm{i}(\phi_1-\phi_2+\chi)}\left\langle\exp\left[\frac{\mathrm{i}}{\hbar c}\int_{-\infty}^{\infty}\mathrm{d}t\int\mathrm{d}\mathbf{r}\,\mathbf{j}_{12}(\mathbf{r},t)\mathbf{A}(\mathbf{r},t)\right]\right\rangle \tag{5}$$

where $\mathbf{j}_{12}(\mathbf{r},t) = \mathbf{j}_1(\mathbf{r},t) - \mathbf{j}_2(\mathbf{r},t)$ and $\langle\cdots\rangle$ means average over $|t_0\rangle$. When the initial state of the environment is an equilibrium state with temperature T, the average means a thermal average $\langle\cdots\rangle_T$.

The second important property of $\mathbf{A}(\mathbf{r},t)$ in the case of no dissipation is that it is a Gaussian operator with respect to thermal averaging $\langle\cdots\rangle_T$. After expanding $\mathbf{A}$ in normal modes this property follows from the relation [9]

$$\langle\exp(\alpha^* a^\dagger - \alpha a)\rangle_T = \exp\left[\tfrac{1}{2}\langle(\alpha^* a^\dagger - \alpha a)^2\rangle_T\right] = \exp\left[-|\alpha|^2\left(n+\tfrac{1}{2}\right)\right] \tag{6}$$

where $a^\dagger$ is the bosonic operator creating a photon in some normal mode, $n = \langle a^\dagger a\rangle_T$ is the occupation number of this mode and α is a complex number. Using the Gaussian properties of $\mathbf{A}$ one can perform the thermal averaging in equation (5) and obtain

$$K = \frac{1}{2(\hbar c)^2}\int \mathrm{d}t\,\mathrm{d}t' \int \mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}'\; j_{12}^{\alpha}(\mathbf{r},t)\, j_{12}^{\beta}(\mathbf{r}',t')\langle A_\alpha(\mathbf{r},t)A_\beta(\mathbf{r}',t')\rangle_T \tag{7}$$

where $\alpha, \beta = x, y, z$. If one defines the thermal correlator

$$\mathcal{D}_{\alpha\beta}(\mathbf{r},\mathbf{r}';t-t') = \tfrac{1}{2}\langle A_\alpha(\mathbf{r},t)A_\beta(\mathbf{r}',t') + A_\beta(\mathbf{r}',t')A_\alpha(\mathbf{r},t)\rangle_T \tag{8}$$

the final result is

$$K = \frac{1}{2(\hbar c)^2}\int \mathrm{d}t\,\mathrm{d}t' \int \mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}'\; j_{12}^{\alpha}(\mathbf{r},t)\, j_{12}^{\beta}(\mathbf{r}',t')\mathcal{D}_{\alpha\beta}(\mathbf{r},\mathbf{r}';t-t'). \tag{9}$$

It was obtained for $T = 0$ in [6] and for $T \neq 0$ in [7]. We derived it in a different way to emphasize the two assumptions under which this result is valid (for classical currents), namely: (i) the commutator of the field operator $\mathbf{A}(\mathbf{r}, t)$ is an *imaginary c-number* and (ii) $\mathbf{A}(\mathbf{r}, t)$ is a *Gaussian quantity with respect to thermal averaging*.

When dissipation is present, the EM environment Hamiltonian includes not only the EM field, but also the electrons in the absorbing bodies and their interaction with the EM field. If the field operator $\mathbf{A}$ is defined as sandwiched by evolution exponents containing the EM field Hamiltonian *only*, it has to be considered as a *random* quantity due to the influence of the dissipative electron system in the absorbing bodies. These electrons are the thermal bath, whose temperature defines the temperature of the EM field. Being a random operator, $\mathbf{A}$ obeys the quantum Langevin equation, where the effect of the dissipative electrons is simulated by a random force. We will show in what follows that the crucial properties of $\mathbf{A}(\mathbf{r}, t)$ used to derive equation (9) are valid also for the random vector potential operator, and hence equation (9) is valid when dissipative bodies are present. Note, that in the case of dissipation normal modes of the EM field do not exist, the EM field cannot be quantized in the usual way, and this is why one is forced to use the Langevin equation approach.

3. Quantum Langevin equation for the EM field

A quantum Langevin equation for the coordinate operator q of a particle moving in potential $V(q)$, derived in [10], can be written in terms of the particle Lagrangian $L = m\dot{q}^2/2 + V(q)$ as

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}(t)} - \frac{\partial L}{\partial q(t)} + \int_{-\infty}^{t} \mathrm{d}t'\,\gamma(t-t')\dot{q}(t') = F(t). \tag{10}$$

The kernel γ is responsible for the 'friction' produced by the environment, which is a thermal bath at temperature T, and the operator $F(t)$ is the random force. The statistical and commutation properties of the random force are defined by the dissipation kernel γ. Namely, $F(t)$ is a Gaussian stationary random process with $\langle F\rangle = 0$ and a correlator

$$\frac{1}{2}\langle F(t)F(t') + F(t')F(t)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega\exp[-\mathrm{i}\omega(t-t')]\hbar\omega\coth\frac{\hbar\omega}{2T}\,\mathrm{Re}\,\gamma(\omega) \tag{11}$$

while the commutator of the random force is an imaginary c-number,

$$[F(t), F(t')] = \frac{2}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\omega\exp[-\mathrm{i}\omega(t-t')]\hbar\omega\,\mathrm{Re}\,\gamma(\omega). \tag{12}$$

(Note, that this Langevin equation is equivalent to the well-known approaches used by Feynman and Vernon [11] and Caldeira and Legget [12]).

Consider the classical Maxwell equations for the long-wave EM field, $\mathrm{rot}\,\mathbf{E} = -\dot{\mathbf{H}}/c$ and $\mathrm{rot}\,\mathbf{H} = \dot{\mathbf{D}}/c + (4\pi/c)\mathbf{j}$, where $\mathbf{D}$ is the displacement given by

$$\mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + \int_{-\infty}^{t} \mathrm{d}t'\, \chi(\mathbf{r}, t - t')\mathbf{E}(\mathbf{r}, t') \tag{13}$$

and $\mathbf{j}$ is the external current density. Note that fields entering the above macroscopic equations are averaged over a volume $\Delta V = (\Delta L)^3$, where ΔL is large compared to all relevant microscopic lengths, but small compared to the wavelength of the EM field. With the gauge $\mathbf{E} = -\dot{\mathbf{A}}/c$ and $\mathbf{H} = \mathrm{rot}\,\mathbf{A}$ the first Maxwell equation is satisfied and the second gives an equation for the vector potential:

$$\frac{1}{c^2}\ddot{\mathbf{A}}(t) + \mathrm{rot}\,\mathrm{rot}\,\mathbf{A}(t) + \frac{1}{c^2}\int \mathrm{d}t'\, \dot{\mathbf{A}}(t')\dot{\chi}(t - t') = \frac{4\pi}{c}\mathbf{j}(t). \tag{14}$$

Starting from the EM field Lagrangian one can prove that this equation can be considered as the quantum Langevin equation for the random field operator $\mathbf{A}$, if $\mathbf{j}$ is the appropriate random force created by the thermal bath of dissipative electrons. ($j_\alpha \Delta V/c$ plays the role of F and $\dot{\chi}\Delta V/4\pi c^2$ plays the role of γ.) The correlator of this force is known from the fluctuation–dissipation theorem [13],

$$\frac{1}{2}\langle j_\alpha(\mathbf{r}, t) j_\beta(\mathbf{r}', t') + j_\beta(\mathbf{r}', t') j_\alpha(\mathbf{r}, t)\rangle = \int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega(t-t')}(j_\alpha(\mathbf{r}) j_\beta(\mathbf{r}'))_\omega \tag{15}$$

with

$$(j_\alpha(\mathbf{r}) j_\beta(\mathbf{r}'))_\omega = \delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}')\frac{\hbar}{8\pi^2}\omega^2 \coth\frac{\hbar\omega}{2T}\,\mathrm{Im}\,\epsilon(\mathbf{r}, \omega). \tag{16}$$

Comparing this correlator with equation (11), we can find from equation (12) the commutator of the random currents, which turns out to be an imaginary c-number,

$$[j_\alpha(\mathbf{r}, t),\, j_\beta(\mathbf{r}', t')] = \delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}')\frac{\hbar}{2\pi^2}\int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega(t-t')}\omega^2\,\mathrm{Im}\,\epsilon(\mathbf{r}, \omega). \tag{17}$$

(The quantum Langevin equation for the EM field was considered also in [14], but in a form not suitable for our problem.)

The retarded Green function, corresponding to equation (14), obeys

$$\frac{1}{c^2}\ddot{\mathcal{G}}_{\alpha\lambda}(\mathbf{r}, \mathbf{r}'; t) + \mathrm{rot}\,\mathrm{rot}_{\alpha\beta}\mathcal{G}_{\beta\lambda}(\mathbf{r}, \mathbf{r}'; t) + \frac{1}{c^2}\int \mathrm{d}t'\, \dot{\mathcal{G}}_{\alpha\lambda}(\mathbf{r}, \mathbf{r}'; t')\dot{\chi}(t - t')$$
$$= -4\pi\delta_{\alpha\lambda}\delta(\mathbf{r} - \mathbf{r}')\delta(t) \tag{18}$$

with the condition $\mathcal{G}(t) = 0$ for $t < 0$. (The rot operators are defined in terms of the antisymmetric tensor $\delta_{\alpha\sigma\beta}$ as $\mathrm{rot}_{\alpha\beta} = \delta_{\alpha\sigma\beta}\nabla_\sigma$ and $\mathrm{rot}\,\mathrm{rot}_{\alpha\beta} = \nabla_\alpha\nabla_\beta - \delta_{\alpha\beta}\nabla^2$.) Using this Green function one can calculate the random field operator

$$A_\alpha(\mathbf{r}, t) = -\frac{1}{c}\int \mathrm{d}t' \int \mathrm{d}\mathbf{r}'\, \mathcal{G}_{\alpha\beta}(\mathbf{r}, \mathbf{r}'; t - t') j_\beta(\mathbf{r}', t'). \tag{19}$$

Two important consequences follow from this relation. First, as the commutator of the currents is an imaginary c-number and the $\mathcal{G}$ is real, the commutator of the random field operators is also an imaginary c-number. Second, as the current is a stationary Gaussian process and $\mathcal{G}$ depends on $t - t'$, same is the random field operator. Since these two properties of the field operator were crucial for deriving the dephasing factor as given by equation (9) for the case of no dissipation, we proved thereby that this result is valid also when dissipation is present.

The last equation allows also the calculation of the correlator and the commutator of the field operator. The correlator is known [15] to be related to the Green function defined by equation (18),

$$\mathcal{D}_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}', t') = \int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega(t-t')}(A_\alpha(\mathbf{r})A_\beta(\mathbf{r}'))_\omega \tag{20}$$

with

$$(A_\alpha(\mathbf{r})A_\beta(\mathbf{r}'))_\omega = -\frac{\hbar}{\pi}\coth\frac{\hbar\omega}{2T}\,\mathrm{Im}\,\mathcal{G}_{\alpha\beta}(\omega;\mathbf{r},\mathbf{r}') \tag{21}$$

where the Green function in the frequency domain is defined as

$$\mathcal{G}_{\beta\lambda}(\mathbf{r},\mathbf{r}';t) = \int\frac{\mathrm{d}\omega}{2\pi}\,\mathrm{e}^{-\mathrm{i}\omega t}\mathcal{G}_{\beta\lambda}(\omega;\mathbf{r},\mathbf{r}'). \tag{22}$$

The definition of $\mathcal{G}(\omega)$ corresponds to that in [16]. The Green function is symmetric, $\mathcal{G}_{\beta\lambda}(\omega;\mathbf{r},\mathbf{r}') = \mathcal{G}_{\lambda\beta}(\omega;\mathbf{r}',\mathbf{r})$, and it follows from the properties of $\epsilon(\mathbf{r},\omega)$ that $\mathcal{G}_{\beta\lambda}(\omega;\mathbf{r},\mathbf{r}')^* = \mathcal{G}_{\beta\lambda}(-\omega;\mathbf{r},\mathbf{r}')$. One can prove the following important integral relation:

$$\frac{\omega^2}{c^2}\int\mathrm{d}\mathbf{r}_0\,\mathrm{Im}\,\epsilon(\mathbf{r}_0,\omega)\mathcal{G}_{\lambda\alpha}(\omega;\mathbf{r},\mathbf{r}_0)\mathcal{G}_{\lambda'\alpha}(\omega;\mathbf{r}',\mathbf{r}_0)^* = -4\pi\,\mathrm{Im}\,\mathcal{G}_{\lambda\lambda'}(\omega;\mathbf{r},\mathbf{r}') \tag{23}$$

which is an obvious generalization to 3D of the 1D relation given in [14]. (It can be proved using the following Green theorem: $\int\mathrm{d}\mathbf{r}\,\phi\,\mathrm{rot}\,\mathrm{rot}_{\alpha\beta}\psi = \int\mathrm{d}\mathbf{r}\,\mathrm{rot}_{\sigma\alpha}\phi\,\mathrm{rot}_{\sigma\beta}\psi$.) Using this integral relation one can obtain equation (21) and also calculate the commutator

$$[A_\alpha(\mathbf{r})_\omega, A_\beta(\mathbf{r}')_{\omega'}] = -\delta(\omega+\omega')\frac{2\hbar}{\pi^2}\,\mathrm{Im}\,\mathcal{G}_{\alpha\beta}(\omega;\mathbf{r},\mathbf{r}'). \tag{24}$$

It is important to note that in deriving equations (21) and (24) with the help of equation (23) one has to assume that the temperature (entering equation (16)) is constant over the whole space (where $\mathrm{Im}\,\epsilon \neq 0$). This is correct in thermal equilibrium, when all absorbing bodies are at the same temperature, and hence equation (21) provides the correlator for the *equilibrium EM field*. But it does not provide the correlator for the *nonequilibrium thermal EM radiation*, when there is radiation energy exchange between bodies with different temperatures. This correlator can also be calculated using equation (16), but the integral over the source point cannot be simplified using equation (23).

4. Dephasing by an infinite thick plate

In what follows we consider a simple geometry when the dephasing body is a half-space $z < 0$ with $\epsilon(\mathbf{r},\omega) = \epsilon(\omega)$ and the two beams move in vacuum $z > 0$ in a plane $z = d$ parallel to the interface. In this geometry the Green function of the EM field is translational invariant in the x, y plane and hence can be presented as follows [16]:

$$\mathcal{G}_{\alpha\beta}(\omega;\mathbf{r},\mathbf{r}') = \int\frac{\mathrm{d}^2k}{(2\pi)^2}\,\mathrm{e}^{\mathrm{i}\mathbf{k}(\mathbf{R}-\mathbf{R}')}g_{\alpha\beta}(\omega,\mathbf{k}|z,z') \tag{25}$$

where $\mathbf{R}$ is the component of $\mathbf{r}$ in the (x,y) plane and $\mathbf{k}$ is a vector in this plane. Because of the special geometry we are interested in the Green function $\mathcal{G}$ for $z = z' = d$ and $\alpha, \beta = x, y$, and will denote $g_{\alpha\beta}(\omega,\mathbf{k}|d,d) \equiv g_{\alpha\beta}(\omega,\mathbf{k})$. Using the explicit expressions for $g_{\alpha\beta}$ given in [16], one can write

$$g_{\alpha\beta}(\omega,\mathbf{k}) = g_t(\omega,k)\left[\frac{k_\alpha k_\beta}{k^2} - \frac{1}{2}\delta_{\alpha\beta}\right] + \frac{1}{2}\delta_{\alpha\beta}g_l(\omega,k) \tag{26}$$

where

$$g_{l,t}(-\omega,k) = g_{l,t}(\omega,k)^* \qquad g_{\alpha\beta}(\omega,-\mathbf{k}) = g_{\alpha\beta}(\omega,\mathbf{k}) = g_{\alpha\beta}(-\omega,\mathbf{k})^*. \tag{27}$$

The $g_{\alpha\beta}(\omega,\mathbf{k})$ are related to the correlator of the tangential components of the electric field in the plane $z = d$. Using equations (27) one can check from equation (21) that for $\alpha, \beta = x, y$

$$(E_\alpha(\mathbf{R})E_\beta(\mathbf{R}'))_\omega = \int\mathrm{d}^2k\,\mathrm{e}^{\mathrm{i}\mathbf{k}(\mathbf{R}-\mathbf{R}')}(E_\alpha E_\beta)_{\omega\mathbf{k}} \tag{28}$$

with

$$(E_\alpha E_\beta)_{\omega\mathbf{k}} = \frac{2\hbar}{(2\pi)^3}\left(\frac{\omega}{c}\right)^2 \coth\frac{\hbar\omega}{2T}(-\mathrm{Im}\, g_{\alpha\beta}(\omega,\mathbf{k})). \tag{29}$$

The classical current in beam 1 is $j_1(\mathbf{r},t) = e\mathbf{v}_1(t)\delta(\mathbf{R}-\mathbf{R}_1(t))\delta(z-d)$, where $\mathbf{R}_1(t)$ is the trajectory of beam 1 and $\mathbf{v}_1(t) = \mathrm{d}\mathbf{R}_1(t)/\mathrm{d}t$ is the electron velocity in this beam. One can present

$$\mathbf{j}_1(\mathbf{r},t) = e\delta(z-d)\int\frac{\mathrm{d}^2k}{(2\pi)^2}\int \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}\mathbf{k}\mathbf{R}}(\mathbf{l}_1)_{\mathbf{k}\omega} \tag{30}$$

where the radiation amplitude is

$$(\mathbf{l}_1)_{\mathbf{k}\omega} = \int\frac{\mathrm{d}t}{2\pi}\mathbf{v}_1(t)\,\mathrm{e}^{\mathrm{i}\omega t-\mathrm{i}\mathbf{k}\mathbf{R}_1(t)}. \tag{31}$$

The relevant frequencies and wave vectors are those of the EM field created by the electron in beam 1. Similar expressions can be written for beam 2. We now introduce into the dephasing integral, equation (9), the correlator $\mathcal{D}$ expressed in terms of $\mathcal{G}$ according to equation (20), and the Fourier expansions of $\mathbf{j}_{1,2}$ and $\mathcal{G}$ according to equations (30) and (25), we find, using equations (27),

$$K = \frac{e^2}{2\hbar c^2}\int_0^\infty\frac{\mathrm{d}\omega}{2\pi}\coth\frac{\hbar\omega}{2T}\int\frac{\mathrm{d}^2k}{(2\pi)^2}\{-\mathrm{Im}\, g_{\alpha\beta}(\omega,\mathbf{k})\}[(l_\alpha)^*_{\mathbf{k}\omega}(l_\beta)_{\mathbf{k}\omega} + \mathrm{c.c.}] \tag{32}$$

where $(\mathbf{l})_{\mathbf{k}\omega} = (\mathbf{l}_1)_{\mathbf{k}\omega} - (\mathbf{l}_2)_{\mathbf{k}\omega}$. The contribution to this integral comes from frequencies and wave vectors which are present simultaneously in the fluctuation spectra $g(\omega,\mathbf{k})$ and the radiation spectra $(\mathbf{l})_{\mathbf{k}\omega}$. Using equation (26) one can rewrite the integral over d^2k as follows:

$$\frac{1}{2\pi}\int_0^\infty \mathrm{d}k\, k\{-2\,\mathrm{Im}\, g_t(\omega,k)\langle|\hat{l}_{\mathbf{k}\omega}|^2\rangle - \mathrm{Im}[g_l(\omega,k)-g_t(\omega,k)]\langle|(\mathbf{l})_{\mathbf{k}\omega}|^2\rangle\} \tag{33}$$

where $\langle\cdots\rangle$ means angular average and $\hat{l}_{\mathbf{k}\omega} = \mathbf{k}(\mathbf{l})_{\mathbf{k}\omega}/k$.

For slow enough electrons one can use the dipole approximation (DA), when the term $\mathbf{k}\mathbf{R}_1(t)$ in equation (31) can be neglected. The condition for this is $\omega \gg kv$, where ω and k are the typical frequency and wave vector of the EM fluctuations contributing to the integral K, and v is the characteristic electron velocity. In this approximation $(\mathbf{l}_1)_{\mathbf{k}\omega} \equiv (\mathbf{l}_1)_\omega$, and $e(\mathbf{l}_1)_\omega$ is the radiating dipole moment. Now $(\mathbf{l})_{\mathbf{k}\omega} = (\mathbf{l}_1)_\omega - (\mathbf{l}_2)_\omega \equiv (\mathbf{l})_\omega$. We substitute $(\mathbf{l})_\omega$ into equation (32) and as a result in the DA the dephasing integral is

$$K = \frac{e^2}{2\hbar c^2}\int_0^\infty\frac{\mathrm{d}\omega}{2\pi}\coth\frac{\hbar\omega}{2T}S(\omega)|(\mathbf{l})_\omega|^2 \tag{34}$$

where

$$|(\mathbf{l})_\omega|^2 = |(l_x)_\omega|^2 + |(l_y)_\omega|^2 \qquad S(\omega) = -\mathrm{Im}\int\frac{\mathrm{d}^2k}{(2\pi)^2}g_l(\omega,k). \tag{35}$$

$S(\omega)$ is related to the average amplitude of the tangential electric field at the distance d from the interface. From equation (28) at $\mathbf{R}' = \mathbf{R}$ one finds

$$\left(E_t^2\right)_\omega = \left(E_x^2+E_y^2\right)_\omega = \frac{\hbar}{\pi}\left(\frac{\omega}{c}\right)^2\coth\frac{\hbar\omega}{2T}S(\omega). \tag{36}$$

It is convenient to represent $K = K_p + K_e$, where the two terms are the contributions to the integral in the equation (33) of the domains, respectively, $k < \omega/c$ and $k > \omega/c$. In the first domain the wave vector component perpendicular to the interface $k_z = \sqrt{(\omega/c)^2 - k^2}$ is real, which means that this domain corresponds to waves propagating perpendicular to the interface (PW), while in the second one k_z is imaginary and it corresponds to evanescent waves (EW).

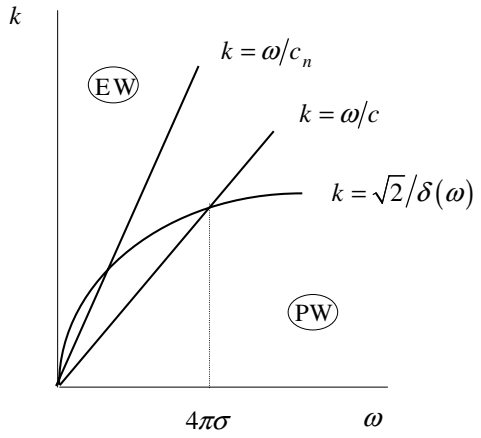

Figure 1. Borderlines in the (ω, k) plane, see text.

5. Spectral densities $g(\omega, k)$

Using the explicit expressions for $g_{\alpha\beta}$ given in [16], one can find (for $\omega > 0$)

$$g_{l,t}(\omega, k) = \frac{2\pi \mathrm{i}}{k_0}[\mathrm{e}^{\mathrm{i}pu(\xi)} F_{l,t}(\xi) - G_{l,t}(\xi)] \tag{37}$$

where $\xi = |\mathbf{k}|/k_0$ with $k_0 = \omega/c$, and

$$G_{l,t}(\xi) = u \pm \frac{1}{u} \qquad F_{l,t}(\xi) = \pm\frac{1}{u}\frac{v-u}{v+u} - u\frac{v-\epsilon(\omega)u}{v+\epsilon(\omega)u} \tag{38}$$

with $u = [1-\xi^2]^{1/2}$, $v = [\epsilon(\omega) - \xi^2]^{1/2}$, $\operatorname{Im} u > 0$, $\operatorname{Im} v > 0$ and $p = 2k_0 d$. The upper sign corresponds to l and the lower to t.

Consider the spectral densities $(-\operatorname{Im} g_{l,t})$ in the (ω, k) plane at $\omega > 0$, $k > 0$ (see figure 1). One important borderline in this plane is $\xi = 1$, i.e. $k = \omega/c$, which, as noted above, separates the propagating waves (PW) domain below it from the evanescent waves (EW) domain above it. In the PW domain $\xi < 1$ and

$$-\operatorname{Im} g_{l,t}(\omega, k) = \frac{2\pi}{k_0}[G_{l,t}(\xi) - \operatorname{Re} F_{l,t}(\xi) \cos pu(\xi)] \tag{39}$$

while in the EW domain $\xi > 1$ and

$$-\operatorname{Im} g_{l,t}(\omega, k) = -\frac{2\pi}{k_0}\,\mathrm{e}^{-p\sqrt{\xi^2-1}} \operatorname{Re} F_{l,t}(\xi). \tag{40}$$

For $\epsilon(\omega) \equiv 1$ one finds $u = v$ and $F(\xi) = 0$. This corresponds to empty space, in which case the spectral densities $-\operatorname{Im} g_{l,t}(\omega, k) \neq 0$ only in the PW domain, where in this case

$$-\operatorname{Im} g_{l,t}(\omega, k) = \frac{2\pi}{k_0} G_{l,t}(\xi). \tag{41}$$

The same result is obviously obtained far from the interface, when $d \to \infty$, since one can neglect the oscillating or decaying term in equation (37).

The second important borderline is $\xi = |\epsilon(\omega)|^{1/2}$. For a non-dispersive lossless dielectric, $\operatorname{Im}\epsilon = 0$, $\epsilon \equiv n^2 > 1$, this borderline is simply $k = \omega/c_n$, where $c_n \equiv c/n$ is the light velocity

in the dielectric. One easily finds that $-\mathrm{Im}\, g_{l,t}(\omega, k) = 0$ above the second borderline, for $\xi > n$. Below it, for $1 < \xi < n$, one has

$$\mathrm{Re}\, F_{l,t}(\xi) = -\frac{2}{\epsilon - 1}(\epsilon - \xi^2)^{1/2}\left[\frac{\epsilon(\xi^2 - 1)}{(\epsilon + 1)\xi^2 - \epsilon} \pm 1\right]. \tag{42}$$

In the generic case of arbitrary complex $\epsilon(\omega)$ the spectral densities $(-\mathrm{Im}\, g_{l,t})$ are nonzero in the whole (ω, k) plane.

In what follows we will consider two cases: (i) a highly polarizable lossless dielectric, $\mathrm{Im}\,\epsilon = 0, \epsilon \gg 1$, and (ii) a 'good' conductor, $\mathrm{Im}\,\epsilon \gg \mathrm{Re}\,\epsilon \simeq 1$. In the last case we write $\epsilon(\omega) = \epsilon_0 + \mathrm{i}(4\pi\sigma/\omega)$, where σ is the conductivity, and assume that σ is larger than all relevant frequencies. Both cases correspond to $|\epsilon| \gg 1$ and it is instructive therefore to investigate the limit $|\epsilon| = \infty$. In this limit $F(\xi) = G(\xi)$ and the spectral densities $-\mathrm{Im}\, g_{l,t}(\omega, k) \neq 0$ only in the PW domain, where

$$-\mathrm{Im}\, g_{l,t}(\omega, k) = \frac{2\pi}{k_0}[1 - \cos pu(\xi)]G_{l,t}(\xi). \tag{43}$$

One can see that near the interface with an ideal conductor ($\sigma = \infty$) or an ideal dielectric ($\epsilon_0 = \infty$) the tangential electric field fluctuations are suppressed. In the case of a conductor it is obvious from the boundary condition $E_t = 0$. In the case of a dielectric this boundary condition is also effective, since $E_t \neq 0$ would mean, due to the continuity of E_t, infinite energy density in the dielectric or infinite displacement current.

Now we turn to the spectral density corrections which are due to finite $|\epsilon|$. To investigate the role of these corrections we use the following expansions. Well below the second borderline, i.e. at $\xi \ll |\epsilon|^{1/2}$, one has

$$F_l(\xi) = G_l(\xi) - 4\epsilon^{-1/2} + O(\epsilon^{-1}) \qquad F_t(\xi) = G_t(\xi)[1 - 2\epsilon^{-1}] + O(\epsilon^{-3/2}). \tag{44}$$

Well above the second borderline, i.e. at $\xi \gg |\epsilon|^{1/2}$, one finds

$$F_{l,t}(\xi) = \mathrm{i}\frac{\epsilon - 1}{\epsilon + 1}\xi + \mathrm{i}\left(\frac{\epsilon - 1}{\epsilon + 1}\right)^2 \frac{1}{2\xi} + O(\xi^{-3}). \tag{45}$$

At $\xi \simeq |\epsilon|^{1/2}$ obviously $F_{l,t}(\xi) \simeq |\epsilon|^{1/2}$.

It follows from equation (44) that in the PW domain the finite $|\epsilon|$ corrections are small. However as we will see later, these corrections are important in the EW domain at small distances from the interface d. To account for these corrections in a dielectric one can use the explicit expressions given by equation (42), but for a conductor the situation is more complicated. For a good conductor it is convenient to use the surface impedance $\zeta(\omega)$ and the skin depth $\delta(\omega)$ defined as follows,

$$\zeta(\omega) = (\omega/8\pi\sigma)^{1/2} \approx (2|\epsilon(\omega)|)^{-1/2} \qquad \delta(\omega) = \frac{c}{(2\pi\sigma\omega)^{1/2}} = 2\zeta(\omega)\frac{c}{\omega}. \tag{46}$$

In these terms the second borderline is $k = \sqrt{2}/\delta(\omega)$ or $\xi = (\sqrt{2}\zeta(\omega))^{-1}$. Since $\zeta(\omega) \ll 1$ and $\delta(\omega) \ll k_0^{-1}$ when $\omega \ll 4\pi\sigma$, this borderline in the EW domain is well above the first borderline $k = \omega/c$.

In between the borderlines, $\omega/c \ll k \ll \delta(\omega)^{-1}$, one can find using equations (44),

$$\mathrm{Re}\, F_l(\xi) = -4\zeta(\omega) \qquad \mathrm{Re}\, F_t(\xi) = -4\zeta(\omega)^2\xi. \tag{47}$$

Above the upper borderline, $k \gg \delta(\omega)^{-1}$, one finds from equation (45), using $(\epsilon - 1)/(\epsilon + 1) = 1 + 4\mathrm{i}\zeta(\omega)^2$, the dominant term to be

$$\mathrm{Re}\, F_{l,t}(\xi) = -4\zeta(\omega)^2\xi. \tag{48}$$

Now we find from equation (40) the spectral densities for a good conductor in the EW domain. In between the borderlines

$$-\mathrm{Im}\, g_l(\omega,k)=\frac{8\pi}{k_0}\zeta(\omega)\mathrm{e}^{-2kd} \qquad -\mathrm{Im}\, g_t(\omega,k)=\frac{8\pi}{k_0^2}\zeta(\omega)^2 k\,\mathrm{e}^{-2kd} \tag{49}$$

while above the upper borderline

$$-\mathrm{Im}\, g_{l,t}(\omega,k)=\frac{8\pi}{k_0^2}\zeta(\omega)^2 k\,\mathrm{e}^{-2kd} \qquad -\mathrm{Im}\, g_{\alpha\beta}(\omega,k)=\frac{8\pi}{k_0^2}\zeta(\omega)^2 k\,\mathrm{e}^{-2kd}\frac{k_\alpha k_\beta}{k^2}. \tag{50}$$

In the EW domain the fluctuations are small, because of the small surface impedance, and are strongly suppressed at $k \gtrsim d^{-1}$, since random fields created by fluctuations of the random currents with wavelength shorter than d are averaged at the distance d.

6. Spectral density $S(\omega)$

The spectral density, which enters in the DA, can be split into contributions of the PW and EW domains, $S(\omega)=S_p(\omega)+S_e(\omega)$, with

$$\begin{aligned} S_p(\omega)&=-\frac{1}{2}k_0\,\mathrm{Re}\int_0^1 \mathrm{d}\xi\,\xi\left[\mathrm{e}^{\mathrm{i}p\sqrt{1-\xi^2}}F_l(\xi)-G_l(\xi)\right]\\ S_e(\omega)&=-\frac{1}{2}k_0\int_1^\infty \mathrm{d}\xi\,\xi\,\mathrm{e}^{-p\sqrt{\xi^2-1}}\,\mathrm{Re}\,F_l(\xi). \end{aligned} \tag{51}$$

To avoid misunderstanding we note that the spectral density due to a half-space $z<0$ with $\epsilon(\omega)$ calculated in [17] is not for the equilibrium EM field, but is for the radiation into a zero temperature half-space $z>0$.

In the limit $|\epsilon|=\infty$ only PW contribute and one finds easily the spectral density

$$S(\omega)=S_p(\omega)=\frac{\omega}{c}\left[\frac{2}{3}-\frac{\cos p}{p^2}-\frac{\sin p}{p}\left(1-\frac{1}{p^2}\right)\right]. \tag{52}$$

At large distances from the interface, $d\gg k_0^{-1}$, one has $S(\omega)=(2/3)k_0$, which corresponds to EM field fluctuations in empty space. (The factor $2/3$ appears because only two tangential components of the electric field are relevant.) At small distances, $d\ll k_0^{-1}$, the fluctuations are suppressed, $S(\omega)\sim d^2$. For an ideal conductor or ideal dielectric the contribution to $S(\omega)$ comes from $\xi\simeq 1$ (i.e. $k\simeq\omega/c$) independent of the value of the parameter $p=2k_0d$.

Now we turn to the finite $|\epsilon|$ corrections. One can see from equation (44) that the corrections to $S_p(\omega)$ are small, and one can use for $S_p(\omega)$ the result given by equation (52). But this is not the case for $S_e(\omega)$ when d is small and the decay of the exponent in this integral is slow. For a dielectric, $S_e(\omega)$ can be calculated using equation (42) and when $\epsilon=n^2\gg 1$ the result is

$$d\ll\lambda_n:\quad S_e(\omega)=\frac{2}{3}k_0 n \qquad d\gg\lambda_n:\quad S_e(\omega)=\frac{1}{2k_0 n}\frac{1}{d^2} \tag{53}$$

where $\lambda_n=c_n/\omega=(k_0 n)^{-1}$ is the wavelength in the dielectric. The first of equations (53) means simply that the fluctuations of E_t at $d\ll\lambda_n$ in the vacuum are the same as in the dielectric, which is consistent with the continuity of E_t at the interface. Equations (53) clearly demonstrate the importance of finite $|\epsilon|$ corrections at small distances. Comparing $S_e(\omega)$ for $d\gg\lambda_n$ with $S_p(\omega)$ from equation (52), one can see that the ideal dielectric approach, when $S(\omega)$ is dominated by PW, is valid only at $d\gg\lambda n^{-1/4}=\lambda_n n^{3/4}$, while at smaller distances $S(\omega)$ is dominated by EW. According to the ideal dielectric approximation, equation (52), near the interface the fluctuations are suppressed compared to free space, while from the first

of equations (53) it follows that they are enhanced compared to free space by a factor n. The contribution to the integral $S_e(\omega)$ comes from $k \simeq \lambda_n^{-1}$, when $d \ll \lambda_n$, and from $k \simeq d^{-1}$ when $d \gg \lambda_n$. When $S_e(\omega)$ dominates, both cases correspond to large imaginary k_z. In other words, the fluctuations contributing to $S_e(\omega)$, are due to near fields localized close to the interface.

The situation in a conductor is different, since from equation (48) one can see that the integral $S_e(\omega)$ diverges at $d = 0$. The near-field fluctuations in the case of a conductor are

$$d \ll \delta(\omega) : S_e(\omega) = \frac{\zeta(\omega)^2}{2k_0^2 d^3} = \frac{\delta(\omega)^2}{8d^3} \qquad d \gg \delta(\omega) : S_e(\omega) = \frac{\zeta(\omega)}{2k_0 d^2} = \frac{\delta(\omega)}{4d^2}. \tag{54}$$

The near-field/far-field crossover point is $d_\times \simeq (c/\omega)\zeta(\omega)^{1/4}$ and for all d the relevant $k \simeq d^{-1}$. When $d \ll \delta(\omega)$ the relevant k are above the borderline $k = \sqrt{2}/\delta(\omega)$ and the first of equations (54) is obtained using equation (50), while when $d \gg \delta(\omega)$ the relevant k are below this borderline and the second of equations (54) is obtained using equation (49).

Comparing $S_e(\omega)$ for a dielectric and a conductor one can see that they are similar not very close to the interface, if one replaces n^{-1} by $\zeta(\omega)$ and λ_n by $\delta(\omega)$. The behaviour very close to the interface is different, since for a dielectric $S_e(\omega)$ is finite, while for a conductor it diverges. (This singularity is cut off if one takes into account the spatial dispersion of the conductor ϵ [17].)

7. Dephasing by near fields

To simplify the picture of dephasing we assume that the electrons smoothly accelerate and decelerate. In other words, we assume that the electron motion has only one characteristic time scale τ, which is the time of flight from the emitter to the detector, and only one length scale L, which is the trajectory length. The characteristic electron velocity is defined as $v = L/\tau$. We also assume, following the experimental situation, that the beams are close, i.e. the distance between them a is small compared to L. The frequencies of the EM waves emitted by the electrons are of the order of $\omega = \tau^{-1}$ and wavelengths are of the order of $\lambda = c\tau$. In this model the angle averages entering the integrals in equations (33) and (34) can be presented as follows:

$$\langle|\hat{I}_{\mathbf{k}\omega}|^2\rangle = \theta L^2 \Psi_1(z, y) \qquad \langle|(\mathbf{I})_{\mathbf{k}\omega}|^2\rangle = \theta L^2 \Psi_2(z, y) \qquad |(\mathbf{I})_\omega|^2 = \theta L^2 \Psi_2(z, 0) \tag{55}$$

where $z = \omega\tau/2$, $y = kL$, and the small factor $\theta = (a/L)^2$ appears because the beams are close and the effective current $\mathbf{j}_{12}$ is smaller than the beam currents $\mathbf{j}_1$ and $\mathbf{j}_2$. The functions Ψ decay fast enough at $\omega \gtrsim \tau^{-1}$ and $k \gtrsim L^{-1}$, restricting the integration in the (ω, k) plane in equation (32) within the rectangle $\square \equiv [0 < \omega \lesssim \tau^{-1}, 0 < k \lesssim L^{-1}]$, shown in figure 2 by dashed lines.

The frequency integral in equation (34) contains three characteristic frequencies, namely, $T/\hbar$ (the frequency of the EM field thermal fluctuations), τ^{-1} (the frequency radiated by the electron) and c/d (the frequency which enters the spectral densities g). Assuming $v/c = 0.1$ and $L = 10$ cm, we have $\tau = 3 \times 10^{-9}$ s. The electro-optical system, which creates, guides and detects the beams, is at room temperature, and this is the temperature of the EM field surrounding the beams. At room temperature $\hbar/T = 2.5 \times 10^{-14}$ s and obviously always $T/\hbar \gg \tau^{-1}$. Hence one can replace in the integral equation (32) $\coth(\hbar\omega/2T)$ by its classical high temperature approximation.

From the spectral densities calculated in sections 5 and 6 it follows that when $|\epsilon| \gg 1$ the contribution K_p of the PW can be calculated as for an ideal conductor or dielectric. To calculate this contribution one can employ the DA, equation (34), because when $|\epsilon| = \infty$ the

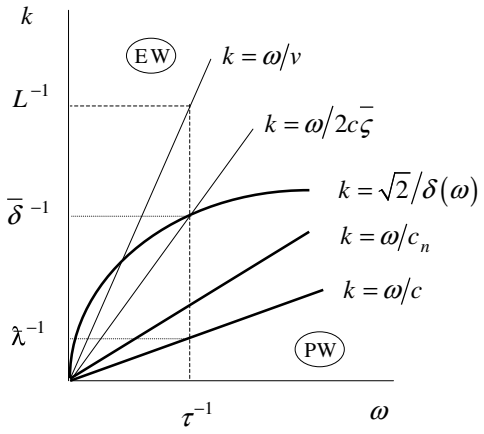

Figure 2. Integration domains in the (ω, k) plane, see text.

density $S(\omega)$ comes from $k \simeq \omega/c$, and for non-relativistic electrons the condition for the DA to be valid, namely $kv \ll \omega$, is satisfied.

First we calculate the dephasing near an ideal conductor or ideal dielectric, given by K_p, equation (34). Substituting there $|(\mathbf{I})_\omega|^2$ from equation (55) we obtain

$$\begin{aligned} d \gg \lambda: \quad & K_p = K_0 = b_{p1}\alpha\theta \left(\frac{L}{c}\right)^2 \frac{T/\hbar}{\tau} \\ d \ll \lambda: \quad & K_p = b_{p2} K_0 \frac{d^2}{\lambda^2}. \end{aligned} \tag{56}$$

Here K_0 is the dephasing in free space, $\alpha = e^2/\hbar c$ is the fine structure constant and the numerical factors $b \simeq 1$ are given in terms of integrals

$$b_{p1} = \frac{2J_0}{3\pi} \qquad b_{p2} = \frac{16J_2}{5J_0} \qquad J_k = \int_0^\infty \mathrm{d}z\, z^k \Psi_2(z, 0). \tag{57}$$

The second of equations (56) demonstrates that near an ideal conductor and dielectric the dephasing is suppressed. For the parameters used one obtains, neglecting numerical factors, $K_0 \simeq 10\theta$. This means that for well-separated beams ($\theta \simeq 1$) at room temperature the dephasing due to thermal fluctuations can be significant. In the existing experiments, however, the beams are very close, $a = 100\,\mu$m, and the dephasing is negligible, $K_0 \simeq 10^{-5}$.

Now we turn to the EW contribution K_e, which is responsible for the enhancement of the dephasing near the interface. As one can see from figure 2, this contribution is large for small electron velocities v, when the overlap of the rectangle $\square$ with the EW domain is maximal. Motivated by this we consider first the simpler case of dielectric for $v \ll c_n = c/n$. Since for a dielectric the spectral densities $-\mathrm{Im}\, g_{l,t}(\omega, k)$ vanish above the borderline $k = \omega/c_n$, the integration domain overlaps only with the 'bottom' of the rectangle $\square$, where k is small, meaning that one can employ the DA. Being interested in an almost ideal dielectric ($n \gg 1$), we substitute the spectral density from equations (53) into equation (34) and find the dephasing due to near fields to be

$$\begin{aligned} d \ll \lambda_n: \quad & K_e = \tfrac{1}{2} K_0 n \\ d \gg \lambda_n: \quad & K_e = b_e K_0 \frac{\lambda^2}{n d^2} \qquad b_e = \frac{3J_{-2}}{8J_0} \end{aligned} \tag{58}$$

where $\lambda_n \equiv c\tau/n$. Comparing the second of equations (58) with equation (56) one finds the far-field–near-field crossover for dephasing to be $d_\times \simeq \lambda n^{-1/4}$, obviously the same as for $S(\omega)$. In contrast to the predictions of the ideal dielectric approximation ($n = \infty$) the dephasing of beams moving near the interface is not suppressed compared to empty space, but enhanced by a large factor $n/2$. Since for most dielectrics n does not exceed 10, the condition $v \ll c/n$ is not very severe for non-relativistic electrons, but on the other hand, the enhancement of the dephasing near the interface is not very strong.

The situation is much more complicated for conductors, since the spectral densities $-\mathrm{Im}\, g_{l,t}(\omega, k)$ do not vanish above the upper borderline $k = \sqrt{2}/\delta(\omega)$ and the integration domain overlaps with the whole rectangle $\square$. The parameter which plays the role of $1/n$ in the case of a conductor is $\overline{\zeta} = (8\pi\sigma\tau)^{-1/2}$, i.e. the surface impedance calculated for the characteristic frequency τ^{-1}. Copper at room temperature has $\sigma = 5 \times 10^{17}\ \mathrm{s}^{-1} = 6 \times 10^5\ (\Omega\ \mathrm{cm})^{-1}$, so for a good conductor this impedance can be as small as 10^{-5}, and hence the restriction $v \ll \overline{\zeta} c$ can be very severe. As a result one has to consider velocities larger than $\overline{\zeta} c$, when the DA might be invalid. Consequently the calculations are very involved, so we first present the results, discuss them and sketch the calculations in the appendix. In what follows we present the results for $\kappa \equiv K/K_0$ and the crossover distance $d_\times$ from near-field to far-field dephasing. The results are given in terms of the trajectory length L, the radiated wavelength $\lambda = c\tau$, the surface impedance $\overline{\zeta}$ and the skin depth $\overline{\delta} = 2\lambda\overline{\zeta}$. All numerical factors of order 1 are omitted.

Three velocity intervals are relevant, namely,

$$A: \quad v/c \ll \overline{\zeta} \qquad B: \quad \overline{\zeta} \ll v/c \ll \overline{\zeta}^{1/4} \qquad C: \quad \overline{\zeta}^{1/4} \ll v/c. \tag{59}$$

The crossover distances from near-field to far-field dephasing in these intervals are as follows:

$$A + B: \quad d_\times = \overline{\zeta}^{1/4}\lambda \qquad C: \quad d_\times = \overline{\zeta}^{1/2}(\lambda^2/L). \tag{60}$$

The far-field dephasing, at $d \gg d_\times$, in all velocity intervals is given by equation (56). The near-field dephasing, at $d \ll d_\times$, is different in different velocity intervals.
In interval A

$$d \ll L: \kappa = \overline{\delta}^2\lambda/L^3 \qquad L \ll d \ll \overline{\delta}: \kappa = \overline{\delta}^2\lambda/d^3 \qquad \overline{\delta} \ll d \ll d_\times: \kappa = \overline{\delta}\lambda/d^2. \tag{61}$$

In interval B:

$$d \ll L: \quad \kappa = \overline{\delta}\lambda/L^2 \qquad L \ll d \ll d_\times: \quad \kappa = \overline{\delta}\lambda/d^2. \tag{62}$$

In interval C:

$$d \ll d_\times: \quad \kappa = \overline{\delta}\lambda/L^2. \tag{63}$$

As one can see from the above results, in the velocity interval $A + B$ the crossover $d_\times$ is the same as for $S(\omega)$ and $L \ll d_\times \ll \lambda$. In this velocity interval K depends on d in a non-monotonic way, reaching a minimum at $d_\times$, where $\kappa \simeq \overline{\zeta}^{1/2}$. In the velocity interval C one finds $\overline{\delta} \ll d_\times \ll L$ and approaching the interface K decays monotonically until $d \simeq d_\times$, where it saturates at $\kappa \simeq \overline{\zeta}(v/c)^{-2}$. For all velocities K is finite at $d = 0$, since at very small d the DA is invalid, and the singularity d^{-3} in $S(\omega)$ is cut off by the ineffectiveness of wave vectors $k \gtrsim L^{-1}$.

As was already mentioned, the dephasing K_0 in empty space is very weak, and this is why the possible enhancement of K near the interface is of special interest. Looking for the ratio $\eta \equiv K(d = 0)/K_0$ one can see from the above results that the dephasing near the interface is enhanced compared to that in empty space only for small enough electron velocities, when $v/c \ll \overline{\zeta}^{1/2}$, i.e. in the interval A and in the smaller velocity part of interval B. Since in the

experiment the fixed parameter is not τ, but L, and hence $\overline{\zeta}$ depends on v, it is more convenient to use a different parameter, namely $\gamma = c/8\pi\sigma L$. In terms of this parameter the velocity interval A is $v/c \ll \gamma$ and the dephasing enhancement in this interval is $\eta = \gamma(v/c)^{-2}$. The smaller velocity part of interval B is $\gamma \ll v/c \ll \gamma^{1/3}$ and here $\eta = \gamma^{1/2}(v/c)^{-3/2}$. (Note also, that the necessary condition $\overline{\zeta} \ll 1$ reduces to $\gamma(v/c) \ll 1$ and is always satisfied when $\gamma \ll 1$.) For the parameters used above one finds $\gamma \simeq 10^{-10}$ and $\gamma^{1/3} \simeq 10^{-3}$. It is clear now that in the case of a good metal the dephasing is enhanced only for relatively slow electrons and is not very high. For example, when $v/c = 10^{-4}$ one finds $\eta \simeq 10$. Much stronger dephasing can be achieved with a high resistivity semiconductor, for example Si with $\sigma = 1\,(\Omega\,\mathrm{cm})^{-1}$, in which case $\gamma \simeq 10^{-4}$ and for $v/c = 10^{-4}$ one finds $\eta \simeq 10^4$.

8. Dissipation versus dephasing

The coherence of the electrons in the beams can be destroyed only if there are mechanisms which allow their energy to be dissipated. When there are no absorbing bodies in the EM environment of the beams, equation (9) describes dephasing related to the dissipation of electron energy by radiation of EM waves 'to infinity'. In fact it means that the energy is dissipated in very far bodies, not included in the consideration explicitly. Equation (9) is formally valid also when the beams are within a lossless cavity, if the small absorption in the walls is still large enough to prevent EM field buildup in the cavity.

If electrons in the two beams move along close trajectories and with similar velocities, $\mathbf{j}_{12} = \mathbf{j}_1 - \mathbf{j}_2$ is small and the dephasing is weak. When the distance between the trajectories is small compared to the correlation length of the EM field in the direction perpendicular to the beams, the random electric fields in adjacent points of the two trajectories fluctuate synchronously, and as a result electrons in both beams change their phases also synchronously, which means that the beams remain mutually coherent. It does not mean, however, that the energy losses in the beams, defined by the currents $\mathbf{j}_1$ and $\mathbf{j}_2$ separately, are small. There is one additional very important difference between dephasing and dissipation. Using the relation $\langle D(\alpha)^\dagger a^\dagger a D(\alpha)\rangle_T = n + |\alpha|^2$, where $D(\alpha) = \exp[\alpha a^\dagger - \alpha^* a]$, one can prove that the energy radiated by a *classical* current into a thermal EM field is

$$W = \frac{1}{2\hbar c^2}\int \mathrm{d}t\,\mathrm{d}t' \int \mathrm{d}\mathbf{r}\,\mathrm{d}\mathbf{r}'\; j^\alpha(\mathbf{r},t)\,j^\beta(\mathbf{r}',t')\mathrm{i}\frac{\partial}{\partial t}[A_\alpha(\mathbf{r},t),A_\beta(\mathbf{r}',t')]. \qquad (64)$$

Hence, in strong contrast to dephasing, the energy losses of the beam electrons do not depend on the environment temperature.

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Appendix

In what follows we sketch the calculations of the results presented in equations (60)–(63). There are two contributions to K_e, namely K_e'', coming from above the borderline $k = \sqrt{2}/\delta(\omega)$, and K_e', coming from between the borderlines $k = \omega/c$ and $k = \sqrt{2}/\delta(\omega)$.

These two contributions can be estimated using $-\mathrm{Im}\, g_{l,t}(\omega, k)$ from equations (50) and (49), respectively. One can also see from figure 2 that when $d \gg L$ the cut off factor e^{-2kd} in $-\mathrm{Im}\, g_{l,t}(\omega, k)$ selects from the rectangle $\square$ only its 'bottom' and hence the DA is valid.

In the velocity interval A the length hierarchy is $L \ll \overline{\delta} \ll \lambda$. When $d \ll \overline{\delta}$ the main contribution is K_e'', where $g_l = g_t$, and the second term in equation (33) vanishes. As a result

$$K_e = K_e'' = \frac{3}{J_0} K_0 \frac{\overline{\delta}^2 \lambda}{L^3} \int_0^\infty \frac{\mathrm{d}z}{z^2} \int_0^\infty \mathrm{d}y\, y^2\, \mathrm{e}^{-(2d/L)y} \Psi_1(z, y). \tag{65}$$

If $d \gg L$ one can put $z = 0$, which corresponds to the DA, in agreement with what was stated above, and obtain

$$K_e = \frac{3J_{-2}}{4J_0} K_0 \frac{\overline{\delta}^2 \lambda}{d^3}. \tag{66}$$

If $d \ll L$ one can put $d = 0$ and the integral is a numerical factor of order 1. When $d \gg \overline{\delta}$ the main contribution is K_e' with g_l dominating, and in addition the DA is valid. Substituting the second of equations (54) into equation (34) one finds

$$K_e = K_e' = \frac{3J_{-3/2}}{2^{7/2} J_0} K_0 \frac{\overline{\delta} \lambda}{d^2}. \tag{67}$$

In the velocity interval $B + C$ the length hierarchy is $\overline{\delta} \ll L \ll \lambda$. Here the main contribution is always K_e' and one can check that g_t can be neglected compared to g_l. As a result

$$K_e = K_e' = \frac{3}{2^{1/2} J_0} K_0 \frac{\overline{\delta} \lambda}{L^2} \int_0^\infty \frac{\mathrm{d}z}{z^{3/2}} \int_0^\infty \mathrm{d}y\, y\, \mathrm{e}^{-(2d/L)y} \Psi_2(z, y). \tag{68}$$

If $d \gg L$ one can put $y = 0$ and obtain

$$K_e = \frac{3J_{-3/2}}{2^{5/2} J_0} K_0 \frac{\overline{\delta} \lambda}{d^2}. \tag{69}$$

When $d \ll L$ one can put $d = 0$ and the integral is a numerical factor. These results are valid in the whole velocity interval $B + C$. The separation appears when one compares the near-field and far-field contributions and finds that $d_\times \gg L$ in B, while $d_\times \ll L$ in C.

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